

Applied Statistical Technique for Portfolio Theory with Application

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Abstract

The summary of this paper is to determine more risky portfolio and safe portfolio of a company for six different assets (GDP/ US, London Gas Oil, FTSE 100, LSE, UK-3 Year Bond, FTSE 350 Beverage) in 2018 over a period of six month, the aim of this data shows the division of first three month and the last three month. Statistical techniques such as (standard deviation, variance, mean, co-variance, expected return, expected risk, probability) were used to conclude the safe and risk portfolio. The results indicate that the safe portfolio for the first three months yield in which it returns to 26.49%, and the risk side returns to 2.25%. This suggest that the selected weights would indeed to be good, but on comparing with the same weights of the second three months of data for the same assets and weight, it was observed that there was a reduction in yield by 23.57% suggesting that some of the assets are needed to be replaced with the risk free assets, to maintain or even increasing the yield.

Keywords: Safe and Risk portfolio, Investment management

المخلص

الفكره الكامله في هذه البحث هو اظهار الطريقتين وهما الطريقه سلبية (الخطرة) و الاخرى الطريقه الايجابية (الامنة) في ستة انواع من الشركات الاجنبية المستثمرة في البلاد في سنة 2018 خلال ستة اشهر و هما:

(GDP/ US, London Gas Oil, FTSE 100, LSE, UK-3 Year Bond, FTSE 350 Beverage)

هذه البيانات المذكورة في النص تبين النتائج في الاشهر الثلاثة الاولى و اشهر الثلاثة أخيرة. الاحصاء التكنيكي مثل (الانحراف المعياري، تباين، التغاير، وسط الحسابي، العائد المتوقع، المخاطر المتوقع، احتمالات) بينت الجانبين السلبي و الايجابي، النتائج أظهرت الجانب الايجابي لثلاثة أشهر الاولى هي 26.49% و الجانب السلبي هي 2.25%. لذلك انه هذه النتائج تبين انها مقارنة جيدة بما فيه الكفاية. و الان اذا نستعمل نفس القياس في الاشهر الثلاثة الاخيرة تظهر النتائج ان نسبة الربح في نفس الشركات تنخفض الى نسبة 23.57% و في النهاية نقترح بارتفاع القوة المستثمرة في الشركات.

پوخته

پوخته له م توژیینه وه دا بۆ دیاری کردنی پۆرتفولیوی مهترسیدار و پۆرتفولیوی پارێزراوه بۆ شهش کۆمپانیای جیاواز له سالی 2018 (GDP/ US, London Gas Oil, FTSE 100, LSE, UK-3 Year Bond, FTSE 350 Beverage) داتاكان وه رگیراوه بۆ شهش مانگی دیاری کراو وه داتاكان دابهش کراوه بۆ دوو بهش هه ر بهشیک داتای سێ مانگی بۆ دیاری ئهکەین یه که بهدای یه که تهکنیکی ئاماری بهکار دێت وه که

(standard deviation, variance, mean, co-variance, expected return, expected risk, probability)

بۆ ديارى كردنى پورتفوليو مەترسیدار و پورتفوليو پارىزراوه. ئەنجامى كۆتای بۆ سى مانگى سەرەتا دەرى ئەخات كە پورتفوليو پارىزراوه پىژەى قازانج دەكاتە 26.49% وە پىژەى مەترسى دەكاتە 2.25%. ئەمەش دەرى ئەخات كە باشترین وەبەرھىنان كردن بۆ ئەو پورتفوليو ئەگەر پىژەى قازانج كە مەترسى ھەيە. ئەگەر ھەمان پىژەر بەكاربھىنین بۆ داتای سى مانگى ئايندە ئەو بۆمان دەرئەكەوێت كە بىرى قازانج بە پىژەى 23.5% دادەبەزىت بەمەش پىژەى پىسك فرى ئەسپتس ئالوگۆر بىكرىت بۆ مەبەستى زىادكردنى بەرھەم.

1. Introduction

According to Kevin (2006) that "the paradox of investing is that more money is better than less", although this statement is generally accepted by all investors. professional portfolio managers can be used for Portfolio management because it requires for time consuming and investment expertise. This case study is based on assessing portfolio theory as an investment tool for making decisions. It is well known that there are various reasons for investments in the world, but the most prominent of all is to earn higher return on an investment. It would never be sufficient to select an investment on the basis of return alone. The very reason that a lot of investors invest much of their funds in many securities gives a suggestion that there may be other factors considered apart from return. This study will thus consider risk as an added factor. Many investors prefer getting returns with minimum risks. In the financial market, apart from the gains obtained, there is a complex volatile industry requiring very critical ways of analysing and evaluating risks and returns. The analysis of risks and returns helps in making decisions on taking part in the industry. This study investigates six months daily historical data for prices of six assets to determine of two portfolio: more risky portfolio and safe portfolio for six different assets (London gas oil, UK-3 year bond, FTSE 100, London stock exchange, FTSE 350 beverage and currency which is pound to US dollar) over a period of the first three month and the last three months, the investments should be taken then examining how the investments performed and how people decide to choose companies to investments.

2. Literature Review

Harry Markowitz 1991, an American economist during the 1950s came up with a theory of "portfolio choice" which help investors to analyze risk and returns of their investments (Comisore et al 2012). Markowitz's theory is today called the Modern Portfolio Theory. It is a theory of investments explaining maximisation of expected returns of a portfolio for some risks. Edwin and Martin (1997) further explained that Modern Portfolio Theory is a complex investment approach which assists investors to select, approximate and control both the type and amount of expected return and expected risk. The critical need to the portfolio is its quantification of relation between risk and return and assuming that there would be some form of compensation to the investor for taking the risk (Yasemin 2012; Comisore et al 2012). According to Choy (2011) is that there is a

constant correlation or fixed between the stocks over a period of time. Similarly, Pajegopal (2012), proved that "it is possible to improve the risk return profile of a portfolio significantly by diversification and suggesting an algorithm for choosing the best portfolio for different investors". The outstanding concept behind the modern portfolio theory is that the selection of assets in an investment should not be on individual basis, but rather it is the great importance to consider relativity on changes of price of assets in the portfolio. This is the reason why this study analyses six assets and evaluates their relative effects on the portfolio based on their daily price.

2.2 Portfolio Theory

According to (Setayesh, 1990) the definition of portfolio theory is that "It is an investment theory based on the idea that risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk, emphasizing that risk is an inherent part of higher reward. It is one of the most important and influential economic theories dealing with finance and investment".

2.3 Concept of expected risk and return

2.3.1 Return

In an investment process, the principal reward is the return which is actually the basic motivating force. The return can be categorised in two forms; the realised return and expected return. Realised return is the return that has been earned and the expected return is the return which the investor looks forward to earn over some period of investment in the future (Benninga 2008). The past returns allow an investor to be able to estimate cash inflow through dividends, capital gains, interest and many other forms. Cosmisore et al (2012) further elaborates that it is thus possible to use return to measure total gains and loss over a period of time. For this research, investment in equity, bond, stock, commodity and currency is analysed on a three month daily prices.

2.3.2 Risk

In investment analysis, risk would be defined as the unpredictability of future returns in an investment. Risk may also be defined as the possibility of having different actual return from expected return (Elton et al 1996). It may further be stated that risk is the possibility of getting a difference in actual outcome of a financial decision from the estimated outcome.

2.4 mean and variance

According to Rajegopal (2012) is that maximize investor returns is normally obtained by constituting of a portfolio. In a Portfolio, going up and down of the actual returns of a group of securities are measured by using of variance and it does this by examining the standard deviations (S.D) of each assets and also it can be used how each assets correlated in the portfolio and the higher of the portfolio variance is made by the higher correlation between the assets in a portfolio. "Mean and variance portfolio construction lead to the capital asset pricing model which is vital in the analysis of investment returns and investment managers use the capital asset pricing model to get the relationship between required return and risk" (Brentani, 2004). According to Lo (2008), Assume investors rely merely on the variance or standard deviation and the expected return of their portfolio: higher variance in bad, higher expected return is good and improve a technique for constructing optimal portfolios.

3. Data collection Techniques

The data have been taken from six different assets (FTSE 100 Index, London Stock Exchange (LSE), London Gas Oil, Great British Pound to United States Dollar (GBP/ USD), FTSE 350 Beaverage, The UK 3-Year Bond through investing web (www.investing.com) for six month daily in 2018. Two groups of the data were taken: The first group taken was to collect the first 3-months daily prices of assets and The period ranged from 2nd January 2018 to 31st March 2018. Then the next 3-months data on daily prices of assets was also collected so that it could be used to check for any differences in prices with the first three months. The dates of the second 3-months data collected ranged from 1st April 2018 to 29th June 2018 the both parts will be compared by using statistical methods (standard deviation, variance, mean, co-variance, expected return, expected risk, probability) to get final conclusion. Moreover, statistical software such as SPSS and Excel will be used to get the final conclusion.

4. Expected Return and Expected Risk

The Expected Return was calculated using the following formula;

$$E(R_p) = \sum_i w_i E(R_i)$$

$E[Return]$: The expected return on the portfolio

n: the number of assets on the portfolio

x_i : The probability of the portfolio invested in asset i

$E[R_i]$: The expected return on asset i

where R_p is the return on the portfolio, and R_i is the return on the asset i . w_i is the weighting of the component asset i . The expected risk can be found by using variance and co-variance as follows:

$$\begin{aligned} V(R_p) &= \text{var}(x_1R_1 + x_2R_2 + x_3R_3 + x_4R_4 + x_5R_5 + x_6R_6) \\ &= x_1^2V_1 + x_2^2V_2 + x_3^2V_3 + x_4^2V_4 + x_5^2V_5 + x_6^2V_6 + 2x_1x_2Cov_{12} + 2x_1x_3Cov_{13} + \dots \end{aligned}$$

$$E[\text{risk}] = \text{standard deviation} = \sqrt{V(R_p)}$$

$V(R_p)$: The variance of asset i

x_i : The probability of the portfolio invested in asset i

Moreover, the risk of return is associated with the variance of the investment return. The variance of the portfolio is thus;

$$V = \text{Var}(R_p) = \text{Var}\left(\sum_i w_i R_j\right) = \sum_i \sum_j w_i R_j \text{Cov}(i, j)$$

The goal is to find the weight which minimizes the risk and maximize the Expected Return. It will be shown with indifference curves how the optimum weights will be selected in the next section of this report (Lo, 2008).

5. Variance and co-variance

Variance and mean were considered in this section on returns to measure the expected value between risky assets. It is clear that the returns of asset follow merely a normal distribution which variance and mean could be a part of normal distribution. Additionally, the following formula can be used in order to find mean and variance.

$$\begin{aligned} E[X] &= \sum_{i=1}^N p_i X_i \\ \text{Var}[X] &= \sum_{i=1}^N p_i [X_i - E(X)]^2 \end{aligned}$$

6. Result and analysis

The summarized data for prices of assets for the first three months will be presented in this section. Firstly the using the daily prices the interest rates of the assets was calculated and is shown in table 1.

Table 1: First three months interest rates

Assets	Interest (%)
GDP/USD	-3.52179
London Gas Oil	0.91719
FTSE 100	7.07891
LSE	23.741
UK 3 Year Bond	6.2599
FTSE 350 Beverage	9.3377

It can be seen from the table that the London Stock Exchange had the highest (23.74%) interest rate for the first three months. The currency exchange rate was lowest with (-3.52%).

In this section, mean and variance were considered on returns to measure the expected utility between risky assets. It is obvious that asset returns follow only a normal distribution which mean and variance can be a part of this description. In addition, mean and variance of portfolio can be found by using the formula below

$$E[X] = \sum_{i=1}^N p_i X_i$$

$$Var[X] = \sum_{i=1}^N p_i [X_i - E(X)]^2$$

Table 2: variance and co-variance between different assets

Assets	GDP/USD	London Gas Oil	FTSE 100	LSE	UK 3 Year Bond	FTSE 350 Beverage
GDP/USD	0.0000147					
London Gas Oil	-0.00000522	0.00010089				
FTSE 100	-0.00000111	0.0000128	0.00004087			
LSE	-0.00001089	0.00000565	0.00005167	0.00020574		
UK 3 Year Bond	-0.00010571	-0.00014067	0.00000934	-0.0000053	0.0102326	
FTSE 350 Beverage	-0.00000055	0.00001166	0.0000274	0.00003417	0.0001027	0.00006205

As shown in the table (2) that variance and co-variance can be found by using statistical techniques due to discover the relationship between assets and to achieve the main object of the study.

variance and co-variance are got by summarizing daily interest rates of the assets and also the co-variance could be used to calculate the risk of assets.

7. Portfolios

Several weights were used to find the best portfolio. Figure 1 show the graph plotted with different weights. It has to be noted that the axes comprise the expected return and the standard deviation. The quadratic indifference curves were used to locate the weights which give the most effective portfolio, that is the portfolio yielding high return but low risk. Furthermore a risk portfolio was also identified.

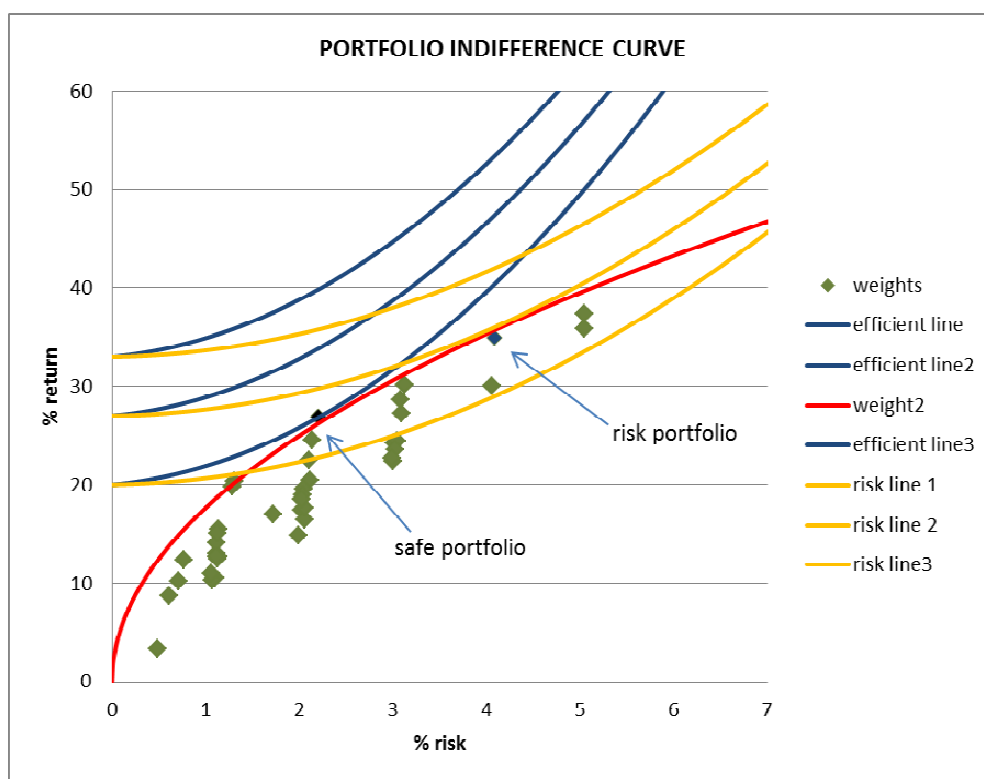


Figure (1): Safe and Risk portfolio

The graph in figure 1 shows the weights and indifference curves. We can therefore summarize the information on the risk and safe portfolio obtained from the graph in the following table:

Table 3: values of risk and safe portfolio

Portfolio for first 3 months	Expected Return (%)	Expected Risk (%)
Safe Portfolio	26.49	2.25
Risky Portfolio	35.59	3.99

It can be seen from table 3 that the best portfolio for the first three months investment in six assets would yield an expected return of 26.29% with 2.25% as the risk. This means it would be worthwhile investing in the assets. and risky portfolio for the first three months investment in six assets would yield an expected return of 35.59% with 3.99% as the risk. As a result, risky portfolio can get more money with more expected risk. However, an analysis of the second three months of the same assets using the same weights will be conducted to see if there will be any significant change in the yield for the same weighting. The actual weights used to obtain the risk and safe portfolio are shown in figure 2.

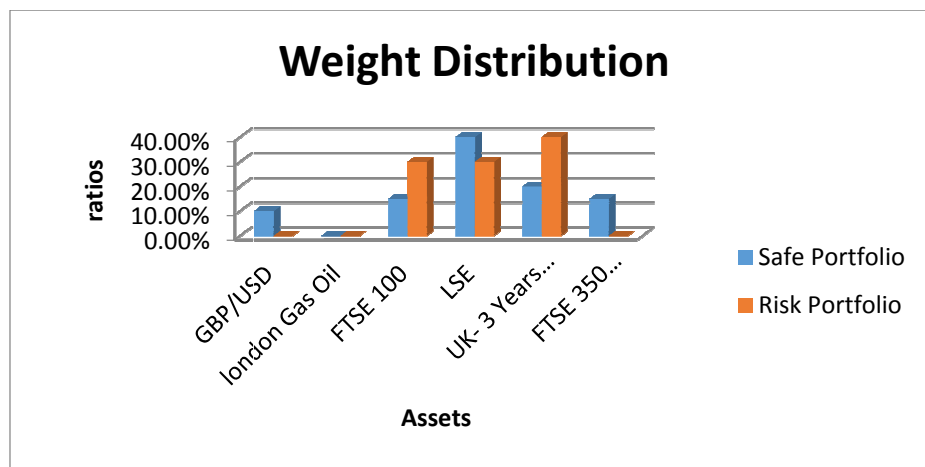


Figure 2: Portfolio weights for the first three months asset data

It can be seen figure 2 that 40% of investment is in London Stock Exchange with the least investment for the safe portfolio being London Gas Oil (0.00%). This is due to the very low interest observed on London Gas oil, thus making it a more risky asset. It would thus be inevitable to replace London Gas Oil with another asset to further optimize the yield of the portfolio.

Table 4 show the changes in interest rate for the six assets between the first three months and the second three months data of daily prices of assets.

Table 4: comparison of three months interest rates.

Assets	Interest (%)	
	First three months	Second three month
GDP/USD	-3.52179	0.71542
London Gas Oil	0.91719	-13.1721
FTSE 100	7.07891	2.86245
LSE	23.741	9.38452
UK 3 Year Bond	6.2599	10.5725
FTSE 350 Beverage	9.3377	5.47215

From the table it can be seen that the average interest for all the six assets reduced in the second three months. This indicate that it may be more risk to invest in the six assets unless the assets with lowest interest such as the London Gas Oil and the currency are replaced with much more stable assets with higher interest.

8. Comparison of Portfolio

The table below shows the changes in the yield and risk in the second three months for weights which produced the safe and risk portfolio in the first three months.

Table 5: Comparison of changes in yield and risk for same weights

Assets	The first three months		The last three months	
	E (Return)	E (Risk)	E (Return)	E (Risk)
Safe Portfolio	26.49	2.25	3.02	1.28
Risky Portfolio	35.59	3.99	8.43	1.49

It can be seen that the yield drastically reduced in the second three months suggesting that investing in all assets without replacing the most risk assets would result in reduced yield. It would therefore be safe to replace the currency and London Gas Oil with a much stable asset.

9. Conclusion

The results indicate that the safe portfolio for the first three months yield a return of 26.49% with risk of 2.25%. This suggest that the weights selected would indeed be good, but on comparing the same weights with the second three months of data for the same assets and weight, it was observed that there was a reduction in yield by 23.57% suggesting that some of the assets needed to be replaced with risk free assets, to maintain or even increase the yield. It would therefore be wise to reconsider the selection of assets for investment before proceeding with investing in the selected six assets.

Finally, it is recommended that although risk portfolio has higher risk and low return, safe portfolio should be chosen as it has higher return and less risk. In other words, risk portfolio has higher return in short term, however, safe portfolio is encouraged due to the long term benefit it has. Safe portfolio is also recommended as the probability of losing the capital is low compared to the high probability of losing the capital in risk investment.

9. References

- Brentani, C. (2003) *Portfolio management in practice*. 2nd Ed. London: Jordan Hill.
- Brentani, C. (2004) *Portfolio Management in Practice*, 2nd Ed. Elsevier Ltd. All rights reserved.
- Benninga, S. (2008), *Financial Modelling* (third edition), MIT press, Massachusetts.
- Choy, M. (2011) *Modern Portfolio Theory using SAS*, text registered OR [online].
[Accessed 7th, Nov. 2018]. Available at <https://arxiv.org/abs/1110.0349>.
- Comisore, L. Yusuf M, Nwofu, I. (2012) The Modern Portfolio Theory as an Investment Decision Tool, *Journal of Accounting and Taxation*, 4(2), pp.(19-28)
- Edwin, E. and Martin, G. (1997). Modern Portfolio Theory, *Journal of bank fin.* pp (1743-1759).
- Elton, E. Gruber J. Stephen, B. and Goetzmann W. (1996). *Modern Portfolio Theory And Investment Analysis*. 8th Ed, John Wiley and Sons, Inc, USA.
- Kevin, S. (2006) *Security analysis and portfolio management*. 2nd Ed. New Delhi: Asoke.
- Lo, A. (2008). *Finance Theory. MIT Sloan Program*.
- Markowitz, H. (1991). Foundations of Portfolio Theory. *The Journal of Finance*. 1 (2). pp. (469-476).
- Pajegopal, S. (2012) *Portfolio management: how to innovate and in successful projects*, 2nd Ed. United Kingdom: Palgrave Macmillan.
- Setayesh, A. (1990). *Modern Portfolio Theory*.
- Yasemin, S. & Idrissi, A. (2012). A Portfolio Approach to Impact Investment. *Global Social Finance*.



10. Appendix

Data

Days	GDP/US	London Gas Oil	FTSE100	LSE	UK-3 Year Bond	FTSE 350 Beverage
2 nd January	1.3507	603.50	7687.77	3797.00	0.494	21737.32
3 rd January	1.3588	598.50	7648.10	3758.00	0.554	21364.42
4 th January	1.3515	610.00	7671.11	3756.00	0.531	21429.90
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28 th June	1.3114	663.25	7621.69	4443.00	0.728	21673.80
29 th June	1.3077	665.50	7615.63	4445.00	0.704	21826.31