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VAR Model System and Impulse Response Function Analysis In Multiple Time Series with Application

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Abstract

This study is to fit and identify a vector autoregressive model system VAR(2)model system for multiple time series,(two time series: the human development indices(HDI) for Iraq, and Iran from (1990 to 2017) are studied here by this type of model which is important and fixable to reach the objectives of this study that's to determine the order of VAR model and recognize the interdependencies among them, and also to evaluate the time length (how many lags time) of dependency they might be continued because the researcher assume and expect as an assumption that this interdependency may continue for a long time because of the similarity of the culture, same religion, commercial relations, geographic location, and social relations between populations in these two countries . Also this study tried to determining the number of lags time cutoff the dependencies (disappearance lag time of dependency) between them, in addition to determining the number of forward time order dependency by adding one positive standard division to the error term for each time series variable above, by using (shock, innovation, or impulse response) to see the impact of each on the other.

Keywords: Vector autoregressive model (VAR), Human development indices (HDI) Impulse response function (IRF), Multiple time series (MTS), Full information maximum likelihood estimates (FIMLE).

الملخص

هذه الدراسة عبارة عن دراسة أحصائية تحليلية تهدف إلى تحديد وتعيين و تقدير منظومة أنموذج (منظومة نماذج (2) VAR) لسلسلة زمنية متعددة, (هناك سلسلتان زمنيتان قيد الدراسة ،هما متسلسلة مؤشرات التنمية البشرية للعراق ، وايران للفترة (من 1990 إلى 2017) ، هذا النوع من الانموذج تم دراسته هنا في هذا البحث مفيدة و يبلغ اهداف الدراسةبسلاسة في تحديد درجة الانموذجVAR و التعرف على الترابط عبر الزمن للمتسلسلتين و تحديد المدى الزمني لامكانية استمرار هذا الترابط هذا الترابط قد تستمر طويلا بسبب تقارب التراث و الدين و التبادلات التجاريةو الموقع الجغرافي للبلدين و العلاقات الاجتماعية للمجتمعين العراقي و الايراني و التي تنشطت اخيرا بعد انقطاع دامت طويلا لاسباب سياسية و عسكرية، كما وان هذه الدراسة حاولت تحديد الفترة الزمنية التي تختفي او تستمر فيها هذه الترابطات، خلال دراسة [دالة استجابة الصدمة لتقييم طول وقت بقاء العلاقة المتبادلة (عدد فترات المتخلفة) من التبعية التي قد تستمر 'طلما العلاقات الاجتماعية والتبادلات التجارية الديانة الاسلامية السمحاء و الحدود الجغرافية ....الخ مستمرة. وكذالك تهدف الدراسة الى تحديد عدد حالات التأخر الزمني في الاعتماد على التبعيات. بالإضافة إلى تحديد عدد فترات التبعية الزمنية الآجلة بإضافة انحراف معياري موجب واحد إلى حجم الخطأ لكل متغير متسلسل زمني أعلى ، باستخدام ما يسمى (الصدمة ، الابتكار ، أو استجابة الاندفاع (impulse, innovation, or shock لرؤية تأثير كل منها على آخر و مدى امكانية استمرارية العلاقة التبادلة في مؤشر التنمية البشرية في البلدين وتاثير احداهما على الاخر.

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پوختە

ئەم تویژینەوەیە بریتیە لە تویژینەوەیەکی شیکاری ئاماری بە ئامانجی دیاری کردن و دروست کردن و خەملاندنی سیستەمێك پێی دەوتریت سیستەمی VAR(2) کە هەڵدەستێت بە مودێل کرنی دوو زنجیرە کات یان زیاتر .دوو زنجیرە کات لەم توێژینەوەیەدا بەکارهاتوە ، زنجیرەی گەشە پێدانی مرۆیی پێوانەیی بو وولاتی عیراق و ئیران، بو سالانی (1990-2017). ئەم جۆرە سستەمە مودێلانە بەکەڵکە بو پێوان و دیاری کردنی پەیوەندیە مروڤایەتیە وابەستەکانی نیوان ئەم دوو وڵاتە لە ڕیگای لیکولینەوە لە فانکشنێك کە پێی دەوترێت فەنکشنی وەلامدانەوەی شۆكimpulse response function ، هەروەها هەڵسەنگاندن و خەملاندنی مەودای کاتی lag time length وابەستەیی گەشە پێدانی مرۆیی و متمانەکردنیان لەسەر یەکتری بەردەوام بوونی ئەم پەیوەندیە ئالّ و گۆرکێییە بەردەوام دەبێت بەهۆی پەیوەندیە کومەلایەتیەکان و گۆڕینەوەی بازرگانی و دینی ئیسلامی پیرۆز کە ئاینی فەرمی هەردوو ولاتەکەن و هاوسنورێتی کە هۆکاری بەردەوام بوونی دەبێت هەرچەندە ئەم پەیوەندیە بۆ ماوەیەکی زۆر توشی لێکبچران بوو بە هۆکاری سیاسی و سەربازی . هەروەها ئەم تویژینەوەیە دیاری کردنی مەودای زەمەنی بەردەوام بوونی ئەم وابەستەییە لە گەشە پێدانی مرۆیی کردوە بە یەکێك لە ئامانجەکانی ئەوەش بە زیادکردنی بارستایی هەڵەی هەڕەمەکیRandom Error بەبڕی یەك لادانی پێوانەیی کە پێی دەوتریت (Shock, Impulse) بو بینینی کاردانەوەی یەکێکیان لەسەر ئەوی تریان و ئەگەری بەردەوام بوونی ئەم وابەستەیە لە زنجیرەی گەشە پێدانی مرۆیی پێوانەیی ئەم دوو وولاتەو کاردانەوەی هەریەکەیان لەسەر ئەوی تر .

1-Introduction:

VAR models are an extension or a generalization of a single autoregressive time series models, VAR model system is a flexible one for multivariate time series data, and econometrics, where the simultaneous equations models specified and identified, this criterion is questioned and advocated as an alternative model firstly by (Sims, C. A. 1980) as VAR models, that criticizing the claims, and performance of earlier modeling in macro-econometrics ,he recommended VAR models, which had previously appeared in time series statistics and in system identification, also in statistical control theory.[13]

AR(P), allowing to more than one evolving time series variable, each has an own explanation equation model, its evolving interested on lagged values for time series itself , and the lagged values of the other model variables, and the residuals or error term, it doesn't require as much information about the strengthen influencing a variable, but only a sufficient knowledge required is a list of variables that assumed to be affective inter-temporally to each other, so VAR model is a stochastic model that can be used to detect and recognize the interdependencies among multiple time series, In other sense if the causality information is available between time series variables involved, for example(y_{1t}, and y_{2t})are two time series variables , assume variable(y_{1t}) is causal for a variable(y_{2t}), then(Granger 1969) defined that VAR models can also be used for analyzing the relation between these involved variables. VAR models also are useful tools for forecasting, if the error term are independent white noise (in VAR model the white noise for each model in the system are uncorrelated, but they are correlated for among the different models in the system) .[3],[9],[13]

The (IRF) is a shock to a VAR system to identify the responsiveness of the dependent variables (endogenous variables), or the origin time series in the system by the changes that may occurs after adding one positive standard division to the error term, which is named by (shock, innovation, or impulse response), and watching to see what may happening for the interdependencies between TS variables in long or short term of time lags and what may occurred on the relation between them.[4],[11]

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2- Methodology:

This section is intended to describe the general ideas of theoretical aspects for VAR models and the algorithms of estimation method in multiple time series analysis, in addition to the IRF's criterion as a tool for detecting interdependencies and the impaction of time series one on each other.

2-1 VAR (p) Model Estimation:

Generally, the best statistical model required the normal distribution or randomness of residuals. Assume that we have derived an estimator over assumption of multivariate normality; then we take the model for the data and obtain and evaluate model estimates under the normality assumption. If the multivariate normality assumption is correct, the residuals should not deviate significantly from the assumption or $\varepsilon_t \sim INp(0,\Psi)$, but if they are auto-correlated or, then the estimates may no longer have optimal properties then the obtained parameter estimates over an incorrectly derived estimator may be meaningless, and since we do not know their 'true' statistical estimation's properties then the model may be under the risk of losing generalization.[4],[5],[12]

2-2 Maximum Likelihood Estimation Method for Unrestricted VAR (P) model:

Before being able to test the assumptions, we need to estimate the model and the following section derives the ML estimator under the null of correct model specification. This section discusses the unrestricted VAR model and illustrates the Maximum likelihood method of estimation in general VAR (P) model.

The general VAR system can be represented by the following form:

$Y_t = \boldsymbol{\Theta}'\boldsymbol{Z}_t + \boldsymbol{\Phi}\boldsymbol{D}_t + \varepsilon_t$, t=1, 2,……,T , and $\varepsilon_t \sim INp(0,\boldsymbol{\Psi})$ ---------- (1)

Where: $\boldsymbol{\Theta}' = \{\boldsymbol{\mu}_0, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_k\}$, $\mathbf{Z}_t = \{1, Y'_{t-1}, Y'_{t-2}, \dots, Y'_{t-k+1}\}$ and the initial values $\boldsymbol{Y^o} = \{Y'_o, Y'_{t-1}, \dots, Y'_{t-k+1}\}$ are given, and $\boldsymbol{D}_t$ is seasonal or dummy affecting matrix for the time series variables. For simplification we ignore the effect of $\boldsymbol{D}_t$ by assuming $\boldsymbol{\Phi}\boldsymbol{D}_t$**=0,** we need to derive the equations for estimating $\boldsymbol{\Theta}$, and $\boldsymbol{\Psi}$ which can be done by finding the expression for $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}$ for which the first order derivatives of the likelihood function are equal to zero. We consider first, the multivariate normal log likelihood function in the following:

$$\ln L(\boldsymbol{\Theta},\boldsymbol{\Psi},\mathbf{Y}) = -T\frac{P}{2}\ln 2\pi - T\frac{1}{2}\ln|\boldsymbol{\Psi}| - \frac{1}{2}\sum_{t=1}^{T}(Y_t - \boldsymbol{\Theta}'\mathbf{Z}_t)'\boldsymbol{\Psi}^{-1}(Y_t - \boldsymbol{\Theta}'\mathbf{Z}_t) \quad \text{---------- (2)}$$

The result of $\frac{\partial \ln L(\boldsymbol{\Theta},\boldsymbol{\Psi},\mathbf{Y})}{\partial \boldsymbol{\Theta}} = \mathbf{0}$, gives: $Y_t\boldsymbol{Z}'_t = \widehat{\boldsymbol{\Theta}}'\sum_{t=1}^{T}\mathbf{Z}_t\mathbf{Z}'_t$ so that the full

Information ML estimator for $\boldsymbol{\Theta}$ is $\widehat{\boldsymbol{\Theta}}' = (\sum_{t=1}^{T} Y_t\,\mathbf{Z}'_t)(\sum_{t=1}^{T}\mathbf{Z}_t\mathbf{Z}'_t)^{-1} = S_{YZ}S^{-1}{}_{ZZ}$

------------ (3)

Next we must calculate $\frac{\partial \ln L(\boldsymbol{\Theta},\boldsymbol{\Psi},\mathbf{Y})}{\partial \boldsymbol{\Psi}} = 0$ **,** then the estimator of $\boldsymbol{\Psi}$ is given by

$$\widehat{\boldsymbol{\Psi}} = \frac{1}{T}\sum_{t=1}^{T}(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t)(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t)' = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t\varepsilon'_t \quad \text{------------ (4)}$$

So by using the equations (3), and (4), we can find the maximum value of the (log) likelihood function for the each maximum likelihood estimates ($\widehat{\boldsymbol{\Theta}}$, and $\widehat{\boldsymbol{\Psi}}$) :

$$\ln L_{max}(\boldsymbol{\Theta},\boldsymbol{\Psi},\mathbf{Y}) = -T\frac{P}{2}\ln 2\pi - T\frac{1}{2}\ln|\widehat{\boldsymbol{\Psi}}| - \frac{1}{2}\sum_{t=1}^{T}(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t)'\widehat{\boldsymbol{\Psi}}^{-1}(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t)$$

---------- (5)

Now we have to show that$\left(\ln L_{max} = -T\frac{1}{2}\ln|\widehat{\boldsymbol{\Psi}}| + \text{constant term}\right)$, then we must consider first that: $(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t)'\widehat{\boldsymbol{\Psi}}^{-1}(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t) = \varepsilon'_t\widehat{\boldsymbol{\Psi}}^{-1}\varepsilon_t = \sum_{i,j}\varepsilon_{t,i}\,\widehat{\boldsymbol{\Psi}}^{-1}\varepsilon_{t,j}$

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$$= \sum_{i,j}(\widehat{\boldsymbol{\Psi}}^{-1})_{i,j}\, \varepsilon_{t,j}\varepsilon_{t,i} == trace(\widehat{\boldsymbol{\Psi}}^{-1}\varepsilon_t\varepsilon_t')$$

Using the result of matrix notation $trace\ AB = \sum_{i,j}(A_{ij}\ B_{ji})$ and summing through (T), we get:

$$\sum_{t=1}^{T}\left(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t\right)\widehat{\boldsymbol{\Psi}}^{-1}\left(Y_t - \widehat{\boldsymbol{\Theta}}'\mathbf{Z}_t\right)' = \sum_{t=1}^{t} trace(\widehat{\boldsymbol{\Psi}}^{-1}\varepsilon_t\varepsilon_t')$$

$$= T\sum_{t=1}^{T} trace(\widehat{\boldsymbol{\Psi}}^{-1}\varepsilon_t\varepsilon_t'/T)$$

$$= T\ trace(\widehat{\boldsymbol{\Psi}}^{-1}\widehat{\boldsymbol{\Psi}}) = T\ trace\left(I_p\right) = T.p$$

Then $\ln L_{max} = -\mathrm{T}\frac{1}{2}\ln\left|\widehat{\boldsymbol{\Psi}}\right| - \mathrm{T}\frac{P}{2} - \mathrm{T}\frac{P}{2}\ln(2\pi)$ this indicates that from the equation (5), the constant term is equal to the quantity $\{-\mathrm{T}(\frac{P}{2}) - \mathrm{T}(\frac{P}{2})\ln(2\pi)\}$, this means that the maximum of the log likelihood function is proportional to the log determinant of the residual covariance matrix ($\widehat{\boldsymbol{\Psi}}$). The derivation of the maximum likelihood estimator for the co-integrated VAR model later. In order to be able to test hypotheses on($\boldsymbol{\Theta}$), we need the distribution of the estimates$\widehat{\boldsymbol{\Theta}}$. Let now we have a case of VAR (p=2) model and we need to discuss the asymptotic distribution of $\widehat{\boldsymbol{\Theta}}$ under the assumption of stationary of the process Y_t, next, consider the estimation error of the VAR (2) coefficients are given by:

$\widehat{\boldsymbol{\Theta}}' - \boldsymbol{\Theta}' = (\widehat{\boldsymbol{\theta}}_1 - \widehat{\boldsymbol{\theta}}_2) - (\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2)$. --------------- (6)

let the variance-covariance matrix between two time lag variables (X_{t-1}, and X_{t-2})

Is given by: $\sum = \begin{bmatrix}\sum_{11} & \sum_{12}\\ \sum_{21} & \sum_{22}\end{bmatrix} = \begin{bmatrix} var(Y_{t-1}) & cov(Y_{t-1}, Y_{t-2})\\ cov(Y_{t-2}, Y_{t-1}) & var(Y_{t-2})\end{bmatrix}$

Under the stationary assumption, the equation (6), has asymptotic normal distribution as:

$\sqrt{T}(\widehat{\boldsymbol{\Theta}} - \boldsymbol{\Theta}) \xrightarrow{asymtotic} N(0, \boldsymbol{\Psi} \otimes \sum^{-1})$ --------------- (7)

such that $\otimes$ is a for kronecker product(If **A** is an $m \times n$ matrix and **B** is a $p \times q$ matrix, then the Kronecker product **A** $\otimes$ **B** is the $mp \times nq$ block matrix).

Where, $\boldsymbol{\Psi} \otimes \sum^{-1} = \begin{bmatrix}\boldsymbol{\Psi}\sum_{11}^{i} & \boldsymbol{\Psi}\sum_{12}^{i}\\ \boldsymbol{\Psi}\sum_{21}^{i} & \boldsymbol{\Psi}\sum_{22}^{i}\end{bmatrix}$, note that the matrix ($\sum$) partitioned in to:

$\sum^{-1} = \begin{bmatrix}\sum_{11}^{i} & \sum_{12}^{i}\\ \sum_{21}^{i} & \sum_{22}^{i}\end{bmatrix}$, i=1, 2.

The asymptotic normal distribution in (7) can be generalized to test any coefficient, now for testing the significant of a such single coefficient, for instance the first element $\theta_{1,11}$ of $\boldsymbol{\theta}_1$, We must define two vectors $\boldsymbol{\alpha}'$ = [1, 0, 0, . . . , 0] and δ' = [1, 0, 0, . . . , 0] where ($\boldsymbol{\alpha}$) is p × 1, and δ is 2p × 1, so that $\boldsymbol{\alpha}'\boldsymbol{\Theta}'\boldsymbol{\delta} = \theta_{1,11}$ Using expression(7), we can find the test statistic for the null hypothesis $\theta_{1,11} = 0$, which has a Normally (0,1) . This can be generalized to testing any coefficient in ($\boldsymbol{\Theta}$), after appropriately choosing the vectors ($\boldsymbol{\alpha}$), and ($\boldsymbol{\delta}$) by the following test.

$$\frac{\sqrt{T}\,\boldsymbol{\alpha}'\boldsymbol{\Theta}'\boldsymbol{\delta}}{(\boldsymbol{\alpha}'\boldsymbol{\Psi}\boldsymbol{\alpha}\boldsymbol{\delta}'\sum^{-1}\boldsymbol{\delta})} \xrightarrow{asymptotic} N(0,1) \qquad \text{-------------- (8)}$$

For a general VAR (p) with (k) variables, with general matrix notation of an identified VAR (1) in two variables ($Y_{1,t}$, $Y_{2,t}$) can be written in matrix form as:

$$\begin{bmatrix}Y_{1,t}\\ Y_{2,t}\end{bmatrix} = \begin{bmatrix}a_1\\ a_2\end{bmatrix} + \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\begin{bmatrix}Y_{1,t-1}\\ Y_{2,t-1}\end{bmatrix} + \begin{bmatrix}\epsilon_{1,t}\\ \epsilon_{2,t}\end{bmatrix} \qquad \text{------------------- (9)}$$

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In which only a single matrix appears because this VAR model has a maximum lag (p) equal to (1), or, equivalently, as the following system of two equations.

$$Y_{1,t} = a_1 + b_{11}Y_{1,t-1} + b_{12}Y_{2,t-1} + \epsilon_{1,t}$$

$$Y_{2,t} = a_2 + b_{21}Y_{1,t-1} + b_{22}Y_{2,t-1} + \epsilon_{2,t} \qquad \text{----------------- (10)}$$

Such that: $Y_{1,t}$, $Y_{2,t}$ are two time series variables.

Each time series in the model has one equation. The current time (t) observation of each variable depends on own lagged values as well as on the lagged values of each other variable in the VAR. This model can be estimated, using MLE approach as discussed above.[2],[5],[8],[10]

2-3 Impulse Response Function (IRF):

IRF is a reaction of any system in response to some extra change (independent variables), then it describes the reaction of the system as a function of time or may be a function of some other independent variables that parameterizes the dynamic behaviors of the system, then its usual to say that the dynamic systems and their IRF's are may be physical objects, or mathematical system of equations describing such objects, since the IRF contains all frequencies, and defines the response of a linear time invariant system for all frequencies, and causing boosting response after low or high frequency. The impulse can be described and modeled, depending on whether the system modeled in discrete or continuous time, and then it can be modeled as the kronecker delta for discrete time system (time series). A system as linear time invariant is completely characterized by its impulse response, that's for any input, the output can be calculated in term of the input and impulse response [output= f (input, and impulse response)in economics, and especially in macroeconomic modeling, IRF's are used to describe the economy reacts through time to exogenous impulses, which economists usually call (shocks), and are often modeled in a VAR model . Impulse response functions describe the reaction of endogenous macroeconomic variables such as output, consumption, investment, and employment at the time of the shock and over subsequent points in time. In linear regression, an exogenous variable is independent of the random error term in the linear model. After a final VAR model was decided, and their parameter values have been estimated, for finding such model holds all variables depend on each other. In order to get a better results for the model's dynamic behaviors, then one can use IRs which they gives us the reaction of a response variable where a one-time shock or innovation was happened. [6],[11]

2-4 Co-integration and Variables Integration Order:

When a collection of time series variables(x_t, y_t, z_t,…., etc) have been studied, they are said to have the (co-integration) statistical property, firstly if all of the series variables must be integrated of order(1), and next, if a linear combination for this collection is (0) order integrated, then the collection is said to be co-integrated.
Let us we write the VAR system with the standard VAR's of infinite error terms as:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y}_t \\ \bar{z}_t \end{bmatrix} + \sum_0^\infty \begin{bmatrix} \boldsymbol{\varphi}_{11}^i & \boldsymbol{\varphi}_{12}^i \\ \boldsymbol{\varphi}_{21}^i & \boldsymbol{\varphi}_{22}^i \end{bmatrix}^i \begin{bmatrix} \varepsilon_{y,t-i} \\ \varepsilon_{z,t-i} \end{bmatrix}$$

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$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y}_t \\ \bar{z}_t \end{bmatrix} + \sum_0^{\infty} \boldsymbol{\Phi}_i \, \varepsilon_{t-i} \qquad \text{--------------------- (11)}$$

Now the impact effect of a one unit positive change in a structural innovation in equation (11), we can get the impact effect of ε_{y_t} on y_t and z_t by taking first derivative for equation (11) as follow:

$\frac{dy_t}{d\varepsilon_{z,t}} = \boldsymbol{\varphi}_{12}(0)$, $\frac{dz_t}{d\varepsilon_{z,t}} = \boldsymbol{\varphi}_{22}(0)$ impact effect for (0) period ahead

$\frac{dy_{t+1,1}}{d\varepsilon_{z,t}} = \boldsymbol{\varphi}_{12}(1)$, $\frac{dz_{t+1}}{d\varepsilon_{z,t}} = \boldsymbol{\varphi}_{22}(1)$ impact effect for (1) period ahead

Note that these impact factors are the same of a structural innovation for one period ago:

$\frac{dy_t}{d\varepsilon_{z,t-1}} = \boldsymbol{\varphi}_{12}(1)$, $\frac{dz_t}{d\varepsilon_{z,t-1}} = \boldsymbol{\varphi}_{22}(1)$, then IRF are the plots of the effect of $(\varepsilon_{z,t})$
----------- (12)

On current and all future of$(y, and\ z)$, and it shows how series$\{y_t\}, and\{z_t\}$ react to each other by the effects of different shocks. So the IRF of (y) to a one unit change in the shock to (z) is given by ($\boldsymbol{\varphi}_{12}(0)$+ $\boldsymbol{\varphi}_{12}(1)$+……+ $\boldsymbol{\varphi}_{12}(i)$) . [6][7][9]

2-5 Variance Decomposition:

The Cholesky (decomposition variant) is the method of choice, for superior efficiency and numerical stability. In econometrics and other applications of multivariate time series analysis, variance decomposition is used to aid in the interpretation of a vector autoregressive (VAR) model once it has been fitted. The (Cholesky-dof) indicates the amount of information each variable contributes to the other variables in the autoregressive for each period. It determines how much of the forecast error variance of each of the variables can be explained by exogenous impulse or shock to the other variables. In practice the effects in equation (12) above cannot be calculated since the structural VAR system is unidentified, then an additional restriction must imposed on the VAR system to identify the IR's , then one can use Cholesky dof adjustment procedure that assumes one of the two time series don't have a recent effect on the second one then the estimate value of the second time series assumed to be zero, which makes the ε_y shocks doesn't affect $z_t$directly but indirectly through the lag effect y_{t-i} in VAR model system.[1][2][4]

3-Application:

Two time series used in this study are about HDI for each (IRAQ, and IRAN) republics as mentioned above. The table below illustrates these two time series. (Eviews statistical package, Version 9 was used during this application).

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3-1 Sample data description:

Table (1): HDI% yearly time series for (IRAQ, and IRAN) , 1990 - 2017

Years	IRAQ.HDI %	IRAN.HDI %	years	IRAQ.HDI %	IRAN.HDI %
1990	0.572	0.577	2012	0.659	0.781
1991	0.527	0.594	2013	0.666	0.784
1992	0.541	0.608	2014	0.666	0.788
1993	0.561	0.620	2015	0.668	0.759
1994	0.561	0.629	2016	0.672	0.796
1995	0.553	0.640	2017	0.685	0.798
1996	0.573	0.647			
1997	0.582	0.653			
1998	0.596	0.659			
1999	0.603	0.664			
2000	0.607	0.670			
2001	0.614	0.678			
2002	0.616	0.683			
2003	0.603	0.689			
2004	0.628	0.691			
2005	0.631	0.695			
2006	0.636	0.731			
2007	0.638	0.736			
2008	0.643	0.741			
2009	0.646	0.747			
2010	0.649	0.755			
2011	0.656	0.766			

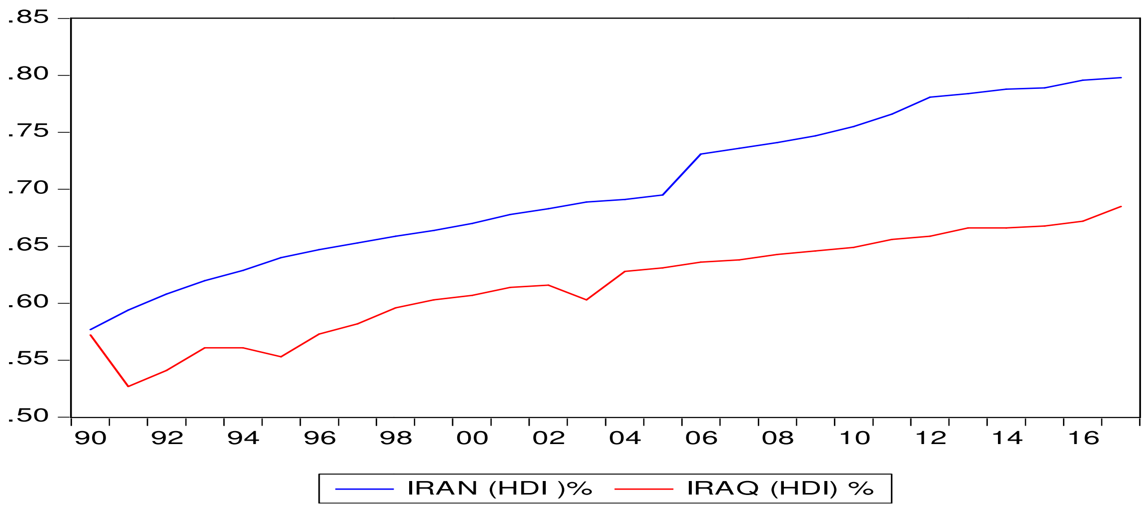

Figure (1): Human Development Indices (%) for Iraq and Iran.

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The Scientific Journal of Cihan University – Sulaimanyia **PP: 47-61**
Volume (3), Issue (1), Jun 2019
ISSN 2520-7377 (Online), ISSN 2520-5102 (Print)

3-2 Unit Root Test:

Table (2): Unit Root Test (Detail).

Endogenous variables: IRAQ.HDI and IRAN.HDI.	
Modulus	Root
0.972970	0.972970
0.875349	0.816438 - 0.315698 i
0.875349	0.816438 + 0.315698 i
0.863758	0.453747 - 0.734978 i
0.863758	0.453747 + 0.734978 i
0.759949	-0.658455 - 0.379421 i
0.759949	-0.658455 + 0.379421 i
0.751615	0.035549 - 0.7507740 i
0.751615	0.035549 + 0.750774 i

The unit root test used for achieving stationary for the two origin Endogenous series, since the all values of modulus are inside the unit circle, indicates the stationary of these time series in the origin, noting that VAR models can't be achieved if stationary is not exist. It is necessary to achieve stationary of endogenous time series before estimating VAR models system so in this study, stationary for both time series were achieved was achieved without differencing, by testing it with unit root test (see table(2)because all the modulus is less than one, means all the roots of the parameters system are inside the unit circle. In order to estimate a VAR (P) model, its required for the series under consideration to be stable, then stability testing must be applied for each two time series, see table(2) for unit root test, since all values of the modulus are inside the unit circle(modulus=$\sqrt{}$ (a^2+ b^2) for the complex number $z = (a+bi)$), then the stationary of these two time series is achieved.

3-3 Estimating VAR (2) model:

Table (3): Estimation values of parameters for VAR (2): The table(3) above shows the estimation for the parameters of the VAR (2) model for two lags time for each, with intercept value(C), note that the value of intercept is non-zero because the two series are originally stationary in mean (without differencing).

Table (4): Estimated VAR(2) model goodness of Fit details:

	IRAQ HDI	IRAN HDI	Variables time lags
Estimate	0.080800	0.598324	IRAQ.HDI$_{t-1}$
S.E	(0.14648)	(0.16466)	
Estimate	0.137317	0.044692	IRAQ.HDI$_{t-2}$
S.E	(0.11947)	(0.13430)	
Estimate	0.858576	0.104246	IRAN.HDI$_{t-1}$
S.E	(0.21225)	(0.23858)	
Estimate	-0.025788	0.105177	IRAN.HDI$_{t-2}$
S.E	(0.21015)	(0.23622)	
Estimate	-0.008760	0.079879	C: intercept
S.E	(0.02861)	(0.03216)	

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The Scientific Journal of Cihan University – Sulaimanyia PP: 47-61
Volume (3), Issue (1), Jun 2019
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The table shows some statistics and tests for the estimated VAR (2) model, R^2= 0.98, and 0.97 for Iraq, and Iran respectively which indicates a best performance and a strong relationship or contribution for each time series with the first and second lag variable for each other. Also the F statistic gave a good indicator for the estimated model performance.

IRAQ.HDI model	IRAN.HDI model	
0.989635	0.970890	R-squared
0.987661	0.965346	Adj. R squared
0.000962	0.001216	Sum sq. residuals
0.006770	0.007609	S.E. equation
501.2825	175.1031	F-statistic
95.76236	92.72196	Log likelihood

VAR (2) Model Formula:

IRAQ.HDI $_t$ =C_1 +ϕ_{11}*IRAQ.HDI $_{t-1}$ +ϕ_{12}*IRAQ.HDI $_{t-2}$ +ϕ_{12}*IRAN.HDI $_{t-1}$ +ϕ_{14}*IRAN.HDI $_{t-2}$ + ε_{t1}

IRAN.HDI $_t$ = C_2 +ϕ_{21}*IRAQ.HDI $_{t-1}$ +ϕ_{22}*IRAQ.HDI $_{t-2}$ +ϕ_{23}*IRAN.HDI $_{t-1}$ +ϕ_{24}*IRAN.HDI $_{t-2}$ + ε_{t2}

------------- (14)

In a matrix notation the Estimated VAR (2) system can be expressed as follow:

$$\begin{bmatrix} IRAQ\ HDI_t \\ IRAN\ HDI_t \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} \end{bmatrix} \begin{bmatrix} IRAQ\ HDI_{t-1} \\ IRAQ\ HDI_{t-2} \\ IRAN\ HDI_{t-1} \\ IRAN\ HDI_{t-2} \end{bmatrix} \quad \text{--------} \ (15)$$

Estimated VAR (2) Model system :

--- :

IRAQ HDI $_{(T)}$ = 0.0798786617718+ 0.598324354649*IRAQ.HDI $_{(t-1)}$ + 0.0446915982332*IRAQ.HDI $_{(t-2)}$ + 0.104245762558*IRAN.HDI $_{(t-1)}$ + 0.105177160862*IRAN.HDI $_{(t-2)}$

IRAN.HDI $_{(T)}$ = - 0.00875964239950 + 0.807995406618*IRAQ.HDI $_{(t-1)}$ + 0.137316815721*IRAQ.HDI $_{(t-2)}$ + 0.858576277555*IRAN.HDI $_{(t-1)}$ - 0.025787875733*IRAN.HDI $_{(t-2)}$

------------------ (16)

The system equation in (17) can be expressed with numerical matrix form as follow:

$$\begin{bmatrix} IRAQ\ HDI_t \\ IRAN\ HDI_t \end{bmatrix} = \begin{bmatrix} 0.07987 \\ -0.00875 \end{bmatrix} + \begin{bmatrix} 0.5983 & 0.0446 & 0.1042 & 0\ .1052 \\ 0{,}8079 & 0.1973 & 0.8585 & -0.0257 \end{bmatrix} \begin{bmatrix} IRAQ\ HDI_{t-1} \\ IRAQ\ HDI_{t-2} \\ IRAN\ HDI_{t-1} \\ IRAN\ HDI_{t-2} \end{bmatrix}$$

----------------- (17)

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Volume (3), Issue (1), Jun 2019
ISSN 2520-7377 (Online), ISSN 2520-5102 (Print)

3-4 VAR (2) Model System Evaluation and Testing:

Prediction Evaluation:

MAPE	MAE	RMSE	Inc. obs.	Variable
12.241	0.0878	0.0906	28	IRAQ.HDI
14.006	0.0879	0.0916	28	IRAN.HDI

Table (5): Predicted and Actual time series with VAR (2) model: shows applying estimated VAR(2) model in equation system (16) on HDI for Iraq, and Iran simultaneously, and model residuals.

Years	IRAQ.HDI	IRAQ.HDI Prediction	IRAN.HDI	IRAN.HDI Prediction	Residuals
1990	0.572	--	0.577	--	-0.058813
1991	0.527	--	0.594	--	0.023949
1992	0.541	0.543368	0.608	0.607482	0.017490
1993	0.561	0.554344	0.620	0.613762	0.000263
1994	0.561	0.563716	0.629	0.621940	0.009263
1995	0.553	0.571326	0.640	0.631065	0.031953
1996	0.573	0.578110	0.647	0.640589	0.009726
1997	0.582	0.584462	0.653	0.650125	0.002574
1998	0.596	0.590561	0.659	0.659511	-0.011885
1999	0.603	0.596476	0.664	0.668689	-0.017115
2000	0.607	0.602231	0.670	0.677643	-0.016960
2001	0.614	0.607838	0.678	0.686371	-0.019190
2002	0.616	0.613301	0.683	0.694876	-0.017113
2003	0.603	0.618626	0.689	0.703166	0.007885
2004	0.628	0.623814	0.691	0.711244	-0.026649
2005	0.631	0.628870	0.695	0.719116	-0.027033
2006	0.636	0.633798	0.731	0.726787	0.001660
2007	0.638	0.638599	0.736	0.734263	0.003737
2008	0.643	0.643279	0.741	0.741549	0.001430
2009	0.646	0.647839	0.747	0.748648	0.003046
2010	0.649	0.652283	0.755	0.755567	0.006662
2011	0.656	0.656614	0.766	0.762310	0.007433
2012	0.659	0.660834	0.781	0.768881	0.018049
2013	0.666	0.664947	0.784	0.775284	0.010819
2014	0.666	0.668955	0.788	0.781524	0.014819
2015	0.668	0.672861	0.789	0.787605	0.012896
2016	0.672	0.676667	0.796	0.793531	0.014051
2017	0.685	0.680377	0.798	0.799306	-0.002947

Table (6): Autocorrelation coefficients for residuals in table (5) and their probability limits after using estimated VAR (2) model in system equations (16): it’s clear that all the values of

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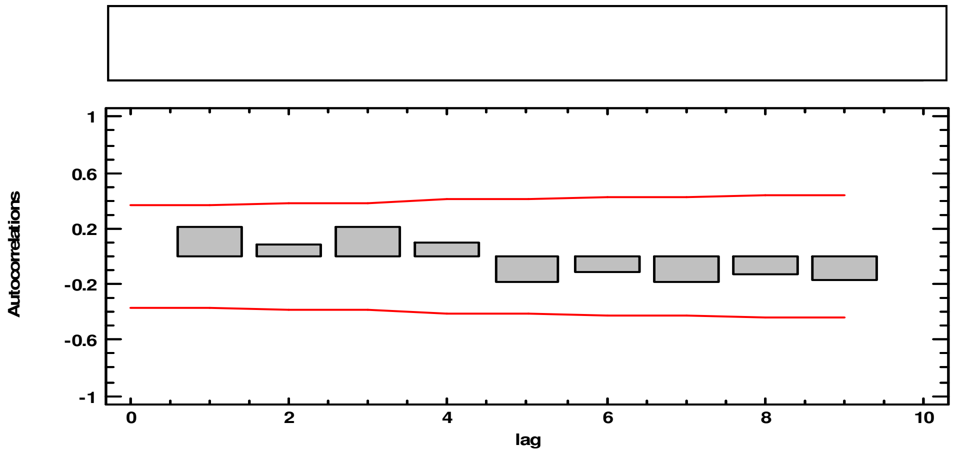
The Scientific Journal of Cihan University – Sulaimanyia PP: 47-61
Volume (3), Issue (1), Jun 2019
ISSN 2520-7377 (Online), ISSN 2520-5102 (Print)

autocorrelation coefficients for residuals in table(5) are inside the %95 confidence interval indicates the randomness for residuals so the estimated VAR(2) model has a good performance.

Upper 95.0%	*Lower 95.0%*			
Prob. Limit	*Prob. Limit*	*Stnd. Error*	*Autocorrelation*	*Lag*
0.370399	-0.370399	0.188982	0.208749	1
0.386203	-0.386203	0.197045	0.0782812	2
0.388373	-0.388373	0.198153	0.214353	3
0.404279	-0.404279	0.206268	0.0993849	4
0.407617	-0.407617	0.207971	-0.185819	5
0.419078	-0.419078	0.213819	-0.118394	6
0.423642	-0.423642	0.216147	-0.19019	7
0.435198	-0.435198	0.222044	-0.120817	8
0.439776	-0.439776	0.224379	-0.16981	9



Figure(2):Estimated Autocorrelation function for residuals after using VAR (2) model, all autocorrelation coefficients falling on %95 c.i indicates a randomness of residuals for estimated model.

Box-Pierce Test based on first 9 autocorrelations, large sample test statistic Q= 6.54302, with P-value = 0.684578 .The Box-Pierce test here is based on the sum of squares of the first 9 autocorrelation coefficients. Since the P-value for this test is greater than or equal to 0.05, we cannot reject the hypothesis that the series is random at the 95.0% or higher confidence level. Then the model system in equations (17) is a more efficient and capable to represent the two time series under consideration, and then the impulse response function acts well to detect the effects or responses for each time series on the other, see figure(3) below which shows the actual and predicted values for time series in the system.

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Volume (3), Issue (1), Jun 2019
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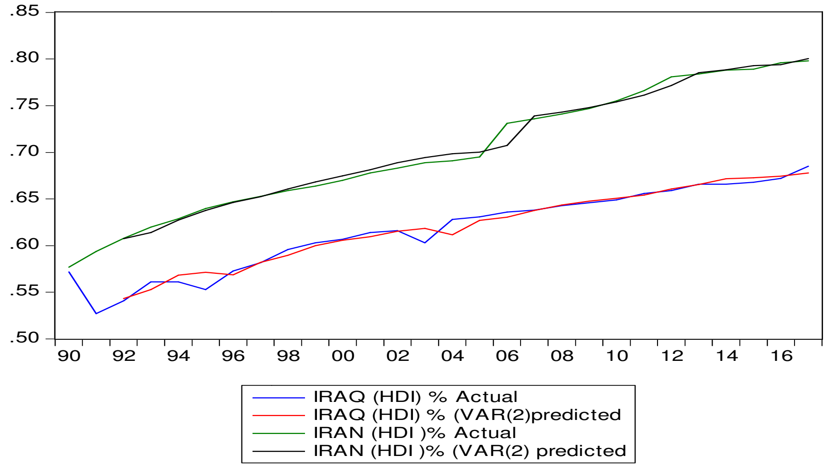

Figure (3): Actual and predicted time series with estimated VAR (2) models system

3-5 Impulse Response Function Analysis:

The figure (4) from below is named by Cholesky dof innovation, which display and explain the interdependencies among two time series as a matrix of graph, by shocking the error term of VAR (2) model system with one standard division, and studding, if the interdependency remain among them for long run time or short run, if it is so at what time lag this relation closing to disappear, here the researcher select (10)time lag to analyze the impulse response for each time series. See figure below.

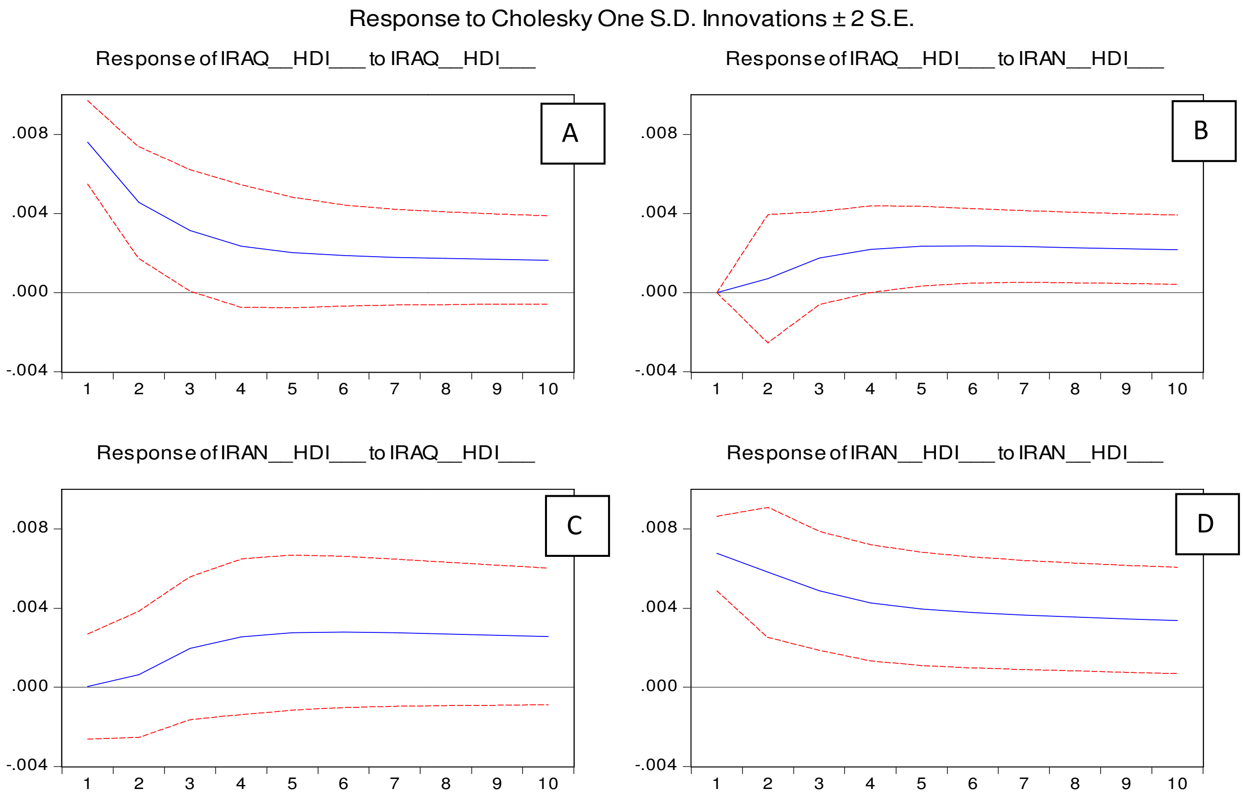

Figure (4): Cholesky dof one standard division impulse shows how the impulse or sock reduce or increase the interdependency for two time series and impacting one on each other until the stability was achieved.

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The Scientific Journal of Cihan University – Sulaimanyia PP: 47-61
Volume (3), Issue (1), Jun 2019
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3-6 Variance Decomposition VAR (2):

Cholesky decomposition details:

Table (7): Variance Decomposition IRAQ.HDI

IRAN.HDI	IRAQ.HDI	S.E.	Period
0.000000	100.0000	0.007609	1
0.629066	99.37093	0.008897	2
3.830968	96.16903	0.009594	3
8.136389	91.86361	0.010119	4
12.35805	87.64195	0.010582	5
16.04598	83.95402	0.011003	6
19.16109	80.83891	0.011389	7
21.78354	78.21646	0.011744	8
24.00689	75.99311	0.012071	9
25.91015	74.08985	0.012373	10

Table (8): Variance Decomposition IRAN.HDI

IRAN.HDI	IRAQ.HDI	S.E.	Period
99.99690	0.003101	0.006770	1
99.47482	0.525178	0.008946	2
96.01229	3.987711	0.010375	3
91.83858	8.161421	0.011506	4
88.16101	11.83899	0.012477	5
85.24605	14.75395	0.013332	6
82.98273	17.01727	0.014096	7
81.20911	18.79089	0.014785	8
79.79340	20.20660	0.015410	9
78.64143	21.35857	0.015982	10

The Cholesky decomposition in the two tables (7,8) is an indicator of an amount of information for each variable contributes to the other variables in the autoregressive for each period. It also determines how much forecast error variance of each time series can be explained by exogenous impulse or shock to the other time series in the system.

4- Conclusions:

From figure (1) one can see clearly that the fluctuations for these two series are similar, which indicates initially that they have the same behaviors and also the unit root test from table (2) indicated that the two series are originally (without taking differencing) stationary.

In order to make sure that the estimated VAR(2) model system(VAR(P) can handle short time series)is an appropriate one and has a good performance, then some concerning tests applied as , sum square error for the system is (0.000962, and 0.001216) for Iraq and Iran HDI respectively and R^2 =(0.989, and 0.971) for them.(see more details from table(4). And figure (3) After fitting VAR(2) models system for Iraq, and Iran HDI and using impulse response function, it is clear that there are mutual influences on each other, the reason may be returned to the fact that the two population follows the Islamic religion. The two cultures are close to each other because of the mutual tirades throughout history, this is in addition to geographical contact and social interaction through marriage and some social closures, then one can say that this interdependency may continue impaction for a long run time as it was proposed by the researcher.

From the results and during applying VAR (2) model on the data, it showed that the model system is more flexible for modeling multiple time series and it is capable to make predictions together (see table 5, and figure 3).and also testing of VAR (2) residual's randomness was made through testing the autocorrelation function by using Box-Pierce test, see table (6), and figure (2).

Concerning to the contribution of each Iraq and Iran HDI in the estimated VAR model, from the tables (7,8) it is clear that the Cholesky decomposition indicated that the two models approximately have the same contribution on the estimated VAR(2) in equations (17), then one can conclude that these two models in the VAR(2) system have an approximate power of influence and impacts on each other and the exogenous impulse or shock for each time series explained clearly and more precise the forecast error variance,

also one can separate each response from figure (4) and conclude that first graph (A), indicates that the response of Iraq human development indices after innovation or shock, is reducing up to five lag time and it attains to stability for higher time lag. The second graph (B), explain the response of Iraq human development and impacting Iran human development on it with an increasing manner starting from zero level at first lag, this effect makes the interdependency and interaction effects going to take stability after lag time(3). The third graph (C), explain the stability of the time lag effects on Iran human development for a long run time. The fourth graph (D), explain the negative impacts for Iraq on Iran human development, the HDI of Iran reduced gradually up to the lag time(6), and then attains the interdependency among them to be stable.

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